EE 230
Lecture 40

Data Converters

Review from Last Time:

Engineering Issues for Using Data Converters

1. Inherent with Data Conversion Process

- Amplitude Quantization
- Time Quantization

(Present even with Ideal Data Converters)

2. Nonideal Components

- Uneven steps
- Offsets
- Gain errors
- Response Time
- Noise

(Present to some degree in all physical Data Converters)

How do these issues ultimately impact performance ?

Review from Last Time:

Time Quantization

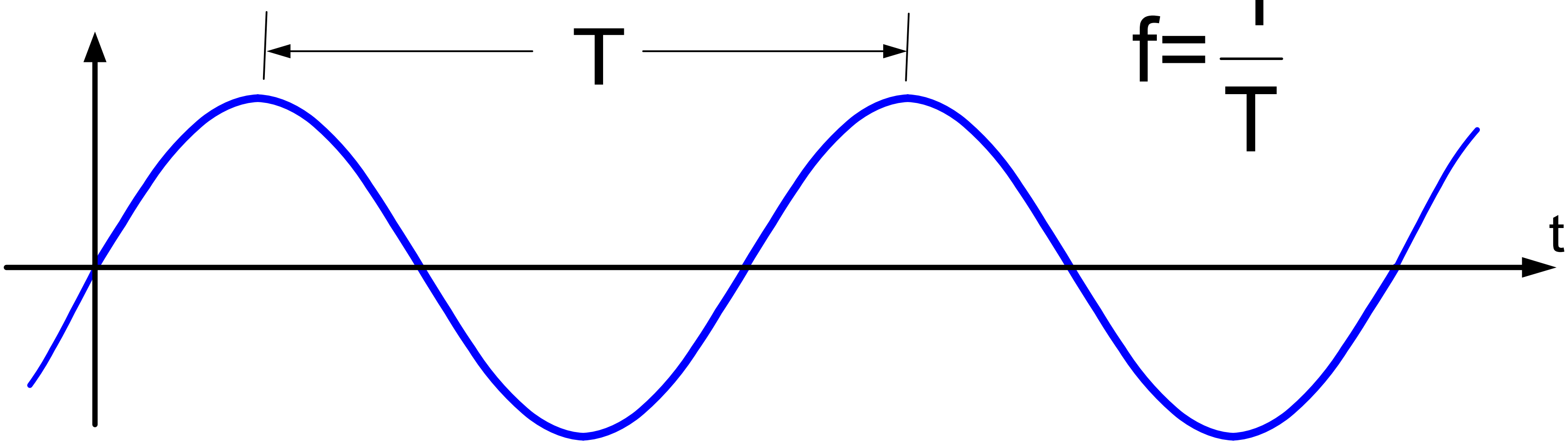
Sampling Theorem

- Aliasing
- Anti-aliasing Filters
- Analog Signal Reconstruction

Review from Last Time:

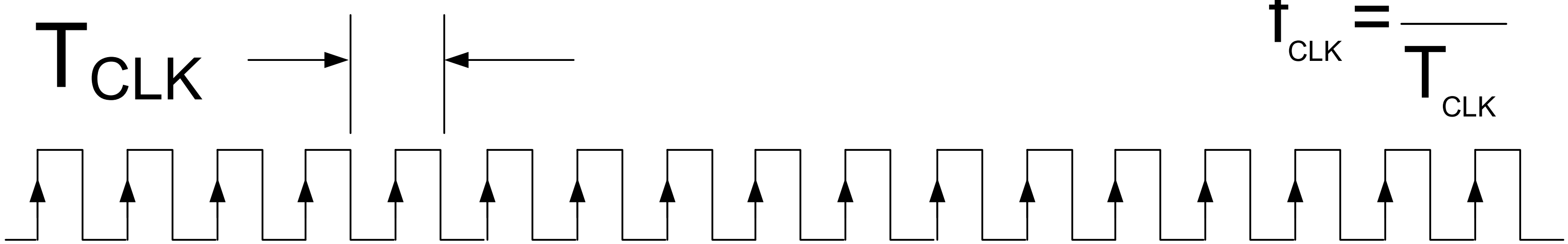
Time Quantization

For convenience, consider a sinusoidal signal



$$f = \frac{1}{T}$$

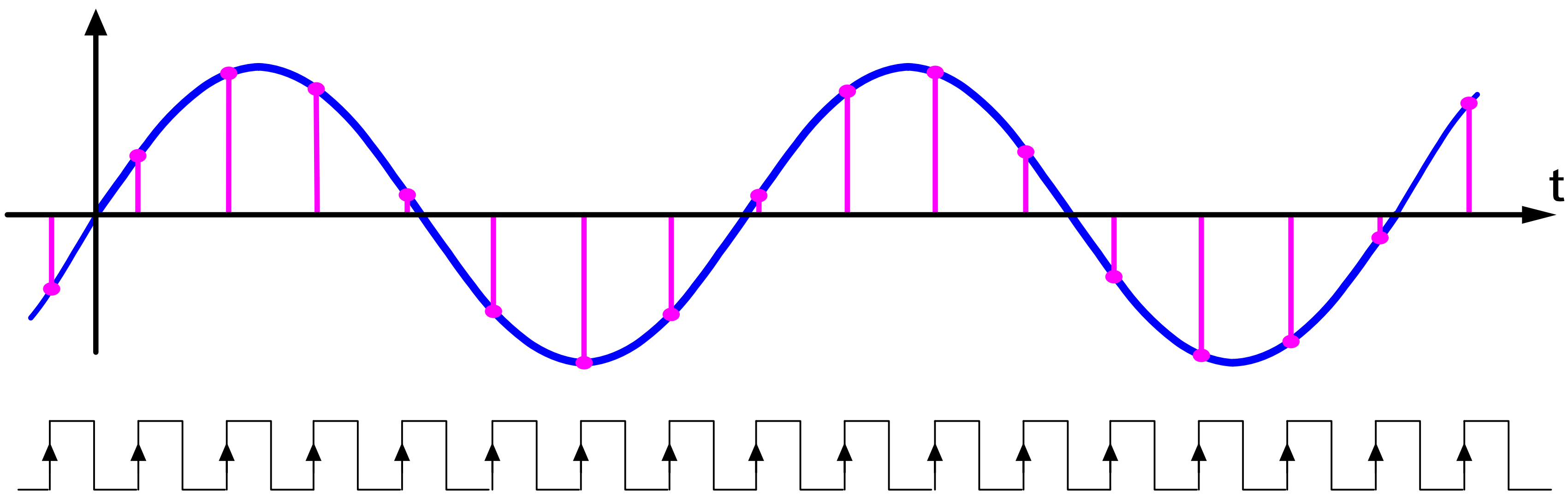
Consider a positive-edge triggered sampling clock signal



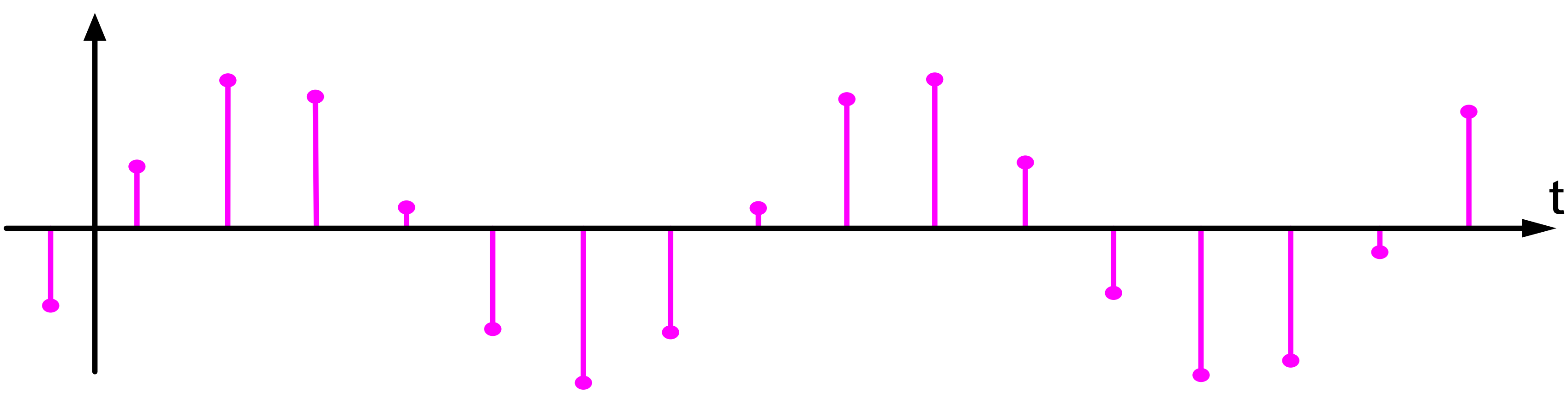
$$f_{CLK} = \frac{1}{T_{CLK}}$$

Review from Last Time:

Time Quantization

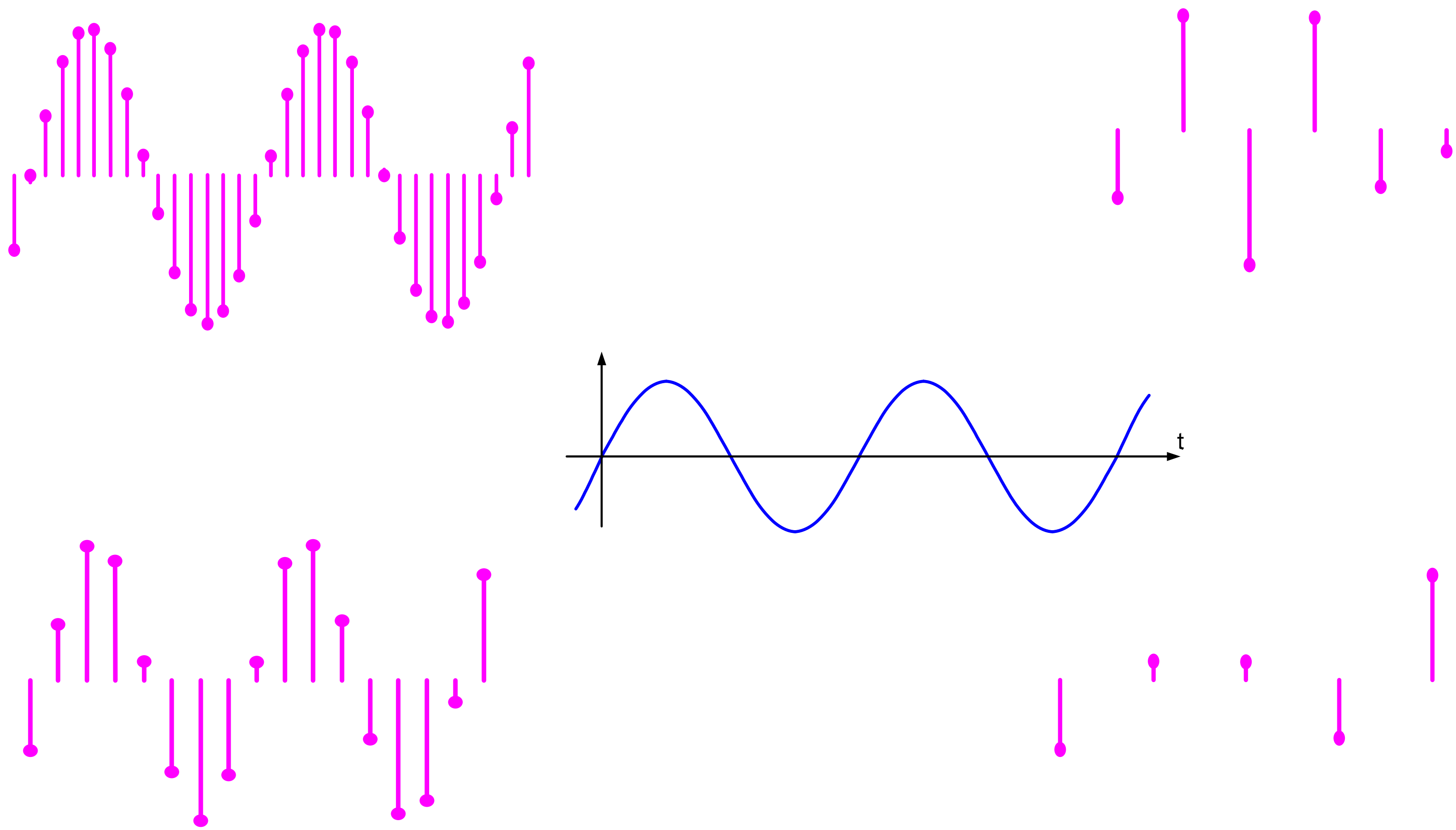


Time-quantized samples of signal



Review from Last Time:

Time Quantization



How often must a signal be sampled so that enough information about the original signal is available in the samples so that the samples can be used to represent the original signal ?

Review from Last Time:

Time Quantization

How often must a signal be sampled so that enough information about the original signal is available in the samples so that the samples can be used to represent the original signal ?

The Sampling Theorem

An exact reconstruction of a continuous-time signal from its samples can be obtained if the signal is band limited and the sampling frequency is greater than twice the signal bandwidth.

This is a key theorem and many existing communication standards and communication systems depend heavily on this property

This theorem often provides a lower bound for clock frequency of ADCs

Sometimes termed Shannon's sampling theorem or the Nyquist-Shannon sampling theorem

Review from Last Time:

Time Quantization

The terms “band limited” and “signal bandwidth” require considerable mathematical rigor to be precise but an intuitive feel for the sampling theorem and the ability to effectively use the sampling theorem can be developed without all of that rigor

The rigorous part:

The Fourier Transform, Y(ω) of a function y(t) is defined as

$$Y(\omega) = \frac{1}{\sqrt{2\pi}} \int_{t=-\infty}^{\infty} y(t) e^{-j\omega t} dt$$

If y(t) is well-behaved (and most functions of interest are), then y(t) can be obtained from Y(ω) from the expression

$$y(t) = \frac{1}{\sqrt{2\pi}} \int_{\omega=-\infty}^{\infty} Y(\omega) e^{j\omega t} d\omega$$

Sometimes termed Shannon’s sampling theorem or the Nyquist-Shannon sampling theorem

Review from Last Time:

Time Quantization

The rigorous part:

$$\mathrm{Y}(\omega)=\frac{1}{\sqrt{2\pi}}\int_{t=-\infty}^{\infty}\mathrm{y}(\mathrm{t})\mathrm{e}^{-j\omega t}\mathrm{dt}$$

Observe the Fourier Transform is very closely related to the Laplace Transform for many (almost all where data converters are used) functions of interest, and they are related by the expression

$$\mathrm{Y}(\omega)=Y(s)\Big|_{s=j\omega}$$

Y(ω) is generally a complex quantity

Review from Last Time:

Time Quantization

The rigorous part:

Signal Bandwidth Definition

If the Fourier Transform of the function $y(t)$ exists and if B is the smallest finite real number for which $Y(\omega)=0$ for all $\omega > B$, then B is the Signal Bandwidth of $y(t)$

Band-limited Definition

If the Fourier Transform of a function $y(t)$ exists, then $y(t)$ is band-limited if there exists a finite real number H such that $Y(\omega)=0$ for all $\omega>H$.

If the signal $y(t)$ is periodic, the sampling theorem can also be given and the concepts of band-limits and signal bandwidth may be more intuitive. This will be discussed later.

Review from Last Time:

Time Quantization

How often must a signal be sampled so that enough information about the original signal is available in the samples so that the samples can be used to represent the original signal ?

The Sampling Theorem

An exact reconstruction of a continuous-time signal from its samples can be obtained if the signal is band limited and the sampling frequency is greater than twice the signal bandwidth.

- the term “band limited” is closely related to term “signal bandwidth”
- the term “Nyquist Rate” in reference to a bandlimited signal is the minimum sampling frequency that can be used if the entire signal can be reconstructed from the samples

$$f_{NYQ}=2B$$

Sometimes termed Shannon’s sampling theorem or the Nyquist-Shannon sampling theorem

Review from Last Time:

Time Quantization

The Sampling Theorem

An exact reconstruction of a continuous-time signal from its samples can be obtained if the signal is band limited and the sampling frequency is greater than twice the signal bandwidth.

Alternatively

An exact reconstruction of a continuous-time signal from its samples can be obtained if the signal is band limited and the sampling frequency exceeds the Nyquist Rate.

Review from Last Time:

Time Quantization

The Sampling Theorem

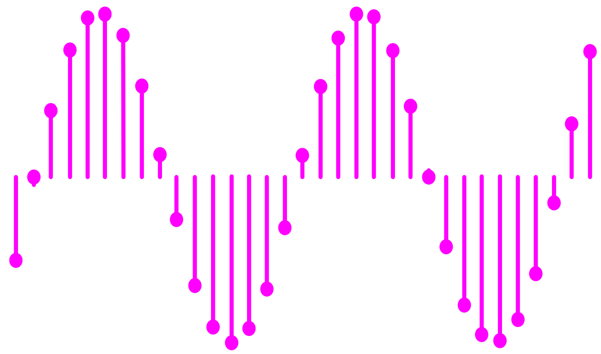
An exact reconstruction of a continuous-time signal from its samples can be obtained if the signal is band limited and the sampling frequency exceeds the Nyquist Rate.

Practically, signals are often sampled at frequency that is just a little bit higher than the Nyquist rate though there are some applications where the sampling is done at a much higher frequency (maybe with minimal benefit)

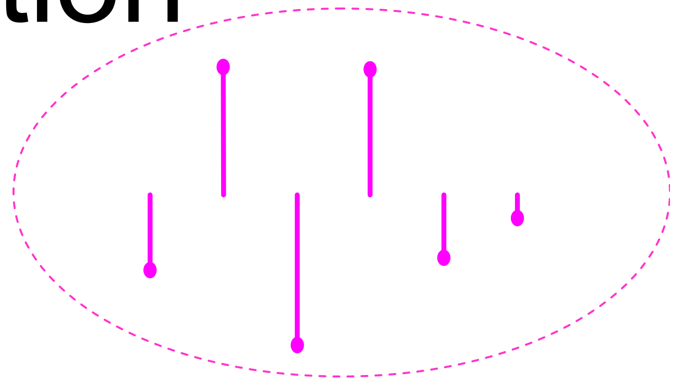
The theorem as stated only indicates sufficient information is available in the samples if the criteria are met to reconstruct the original continuous-time signal, nothing is said about how this can be practically accomplished.

Review from Last Time:

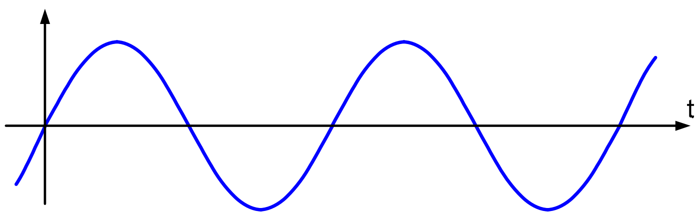
Time Quantization

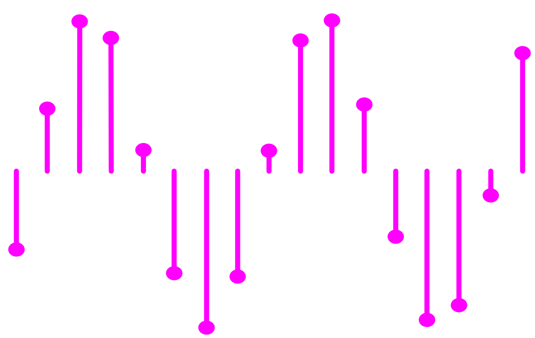


Approximately 6 times f_{NYQ}

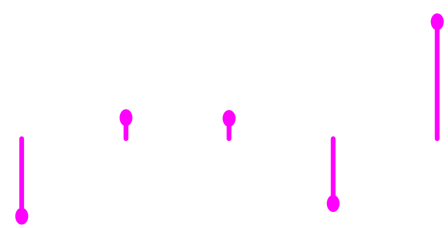


Slightly above f_{NYQ}





Approximately 3 times f_{NYQ}



Below f_{NYQ}

Review from Last Time:

Time Quantization

The Sampling Theorem

An exact reconstruction of a continuous-time signal from its samples can be obtained if the signal is band limited and the sampling frequency exceeds the the Nyquist Rate.

What happens if the requirements for the sampling theorem are not met?

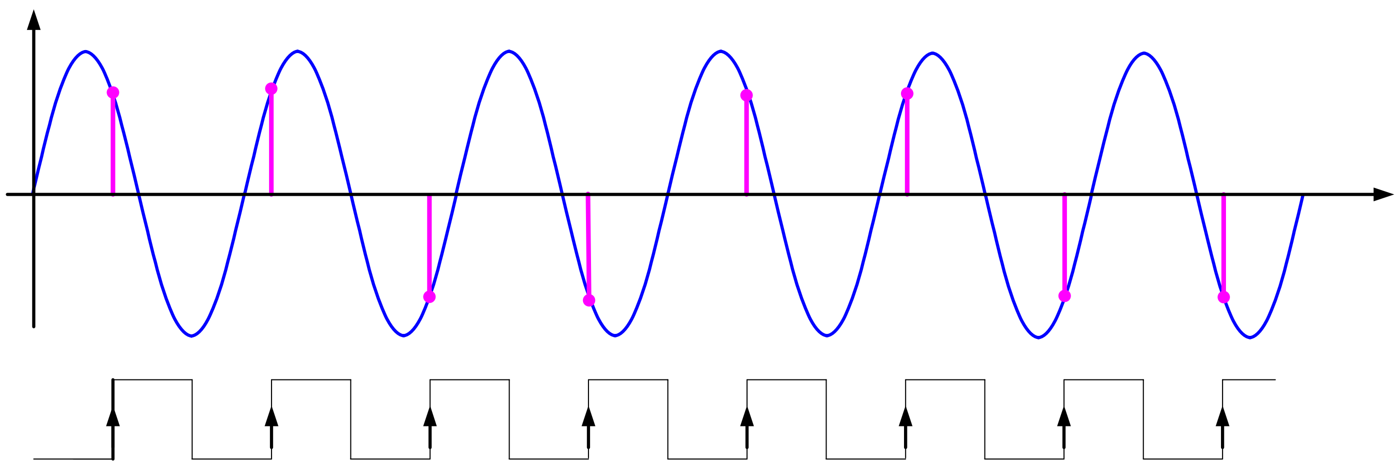
How can a continuous-time signal be practically reconstructed from the samples if the hypothesis of the sampling theorem was satisfied when the samples were taken?

Review from Last Time:

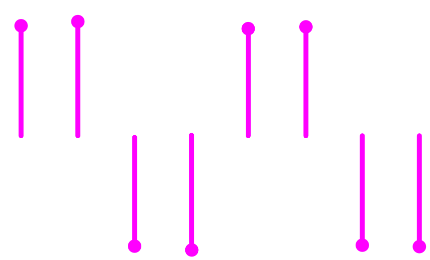
Time Quantization

What happens if the requirements for the sampling theorem are not met?

Example: Consider a signal that is of frequency 3/4 f_{CLK}
Signal violates the hypothesis of the sampling theorem, it is higher in frequency than ½ f_{CLK}



Sampled output sequence:

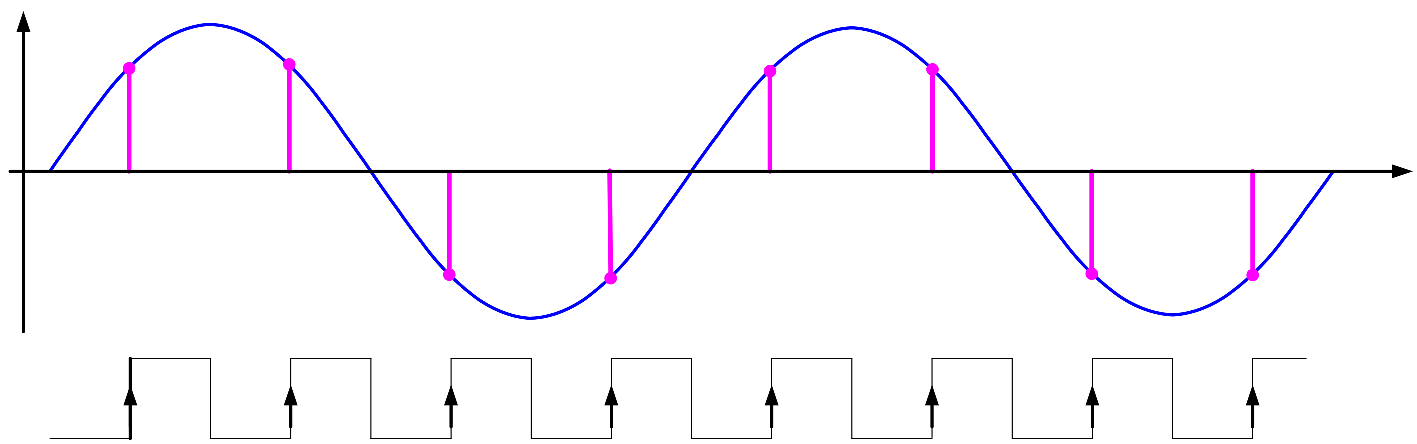


Review from Last Time:

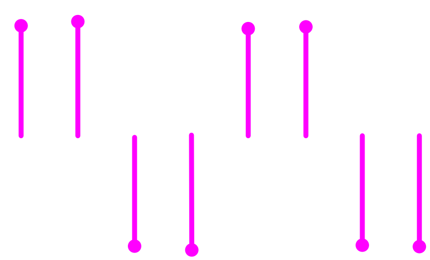
Time Quantization

What happens if the requirements for the sampling theorem are not met?

Example: Consider a signal that is of frequency 1/4 f_{CLK} - assume f_{CLK} same as before
Signal violates the hypothesis of the sampling theorem, it is higher in frequency than ½ f_{CLK}



Sampled output sequence:



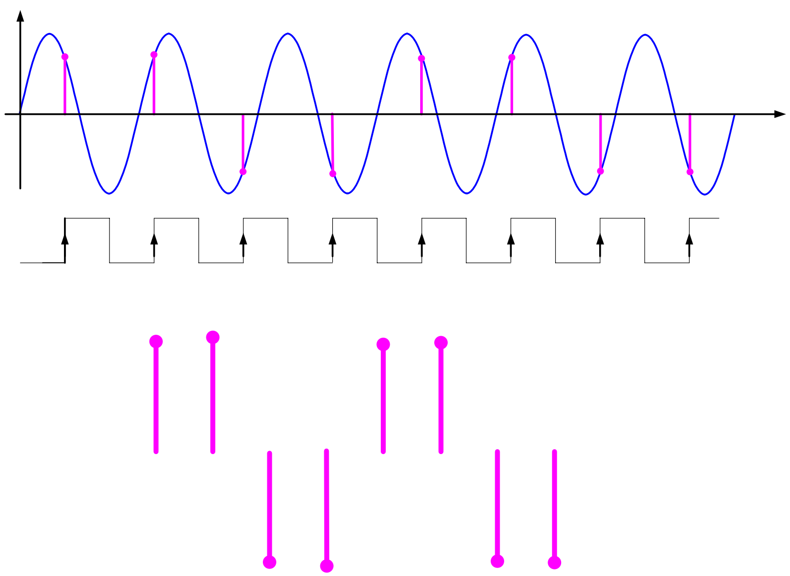
Review from Last Time:

Time Quantization

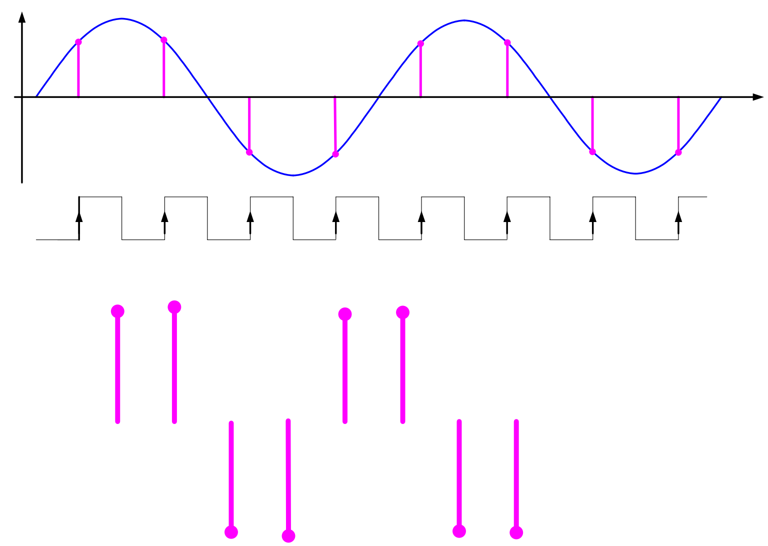
What happens if the requirements for the sampling theorem are not met?

Example:

$f_{SIG}=3/4\ f_{CLK}$



$f_{SIG}=1/4\ f_{CLK}$



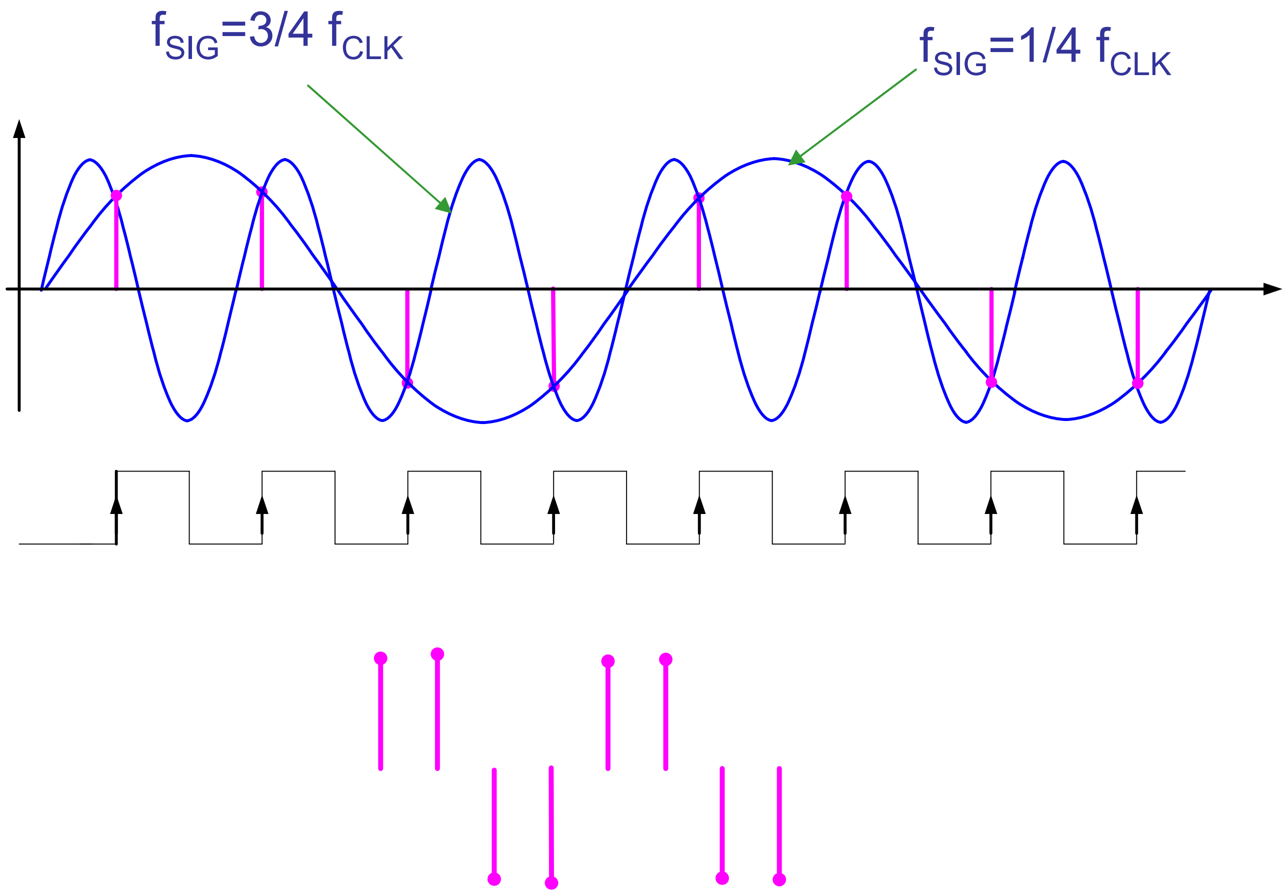
Output sampled sequences are identical!

Review from Last Time:

Time Quantization

What happens if the requirements for the sampling theorem are not met?

Example:



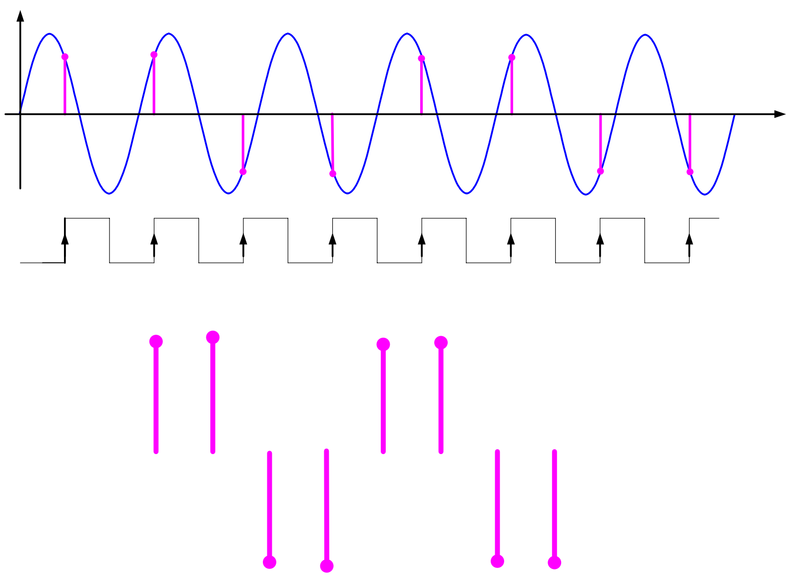
Review from Last Time:

Time Quantization

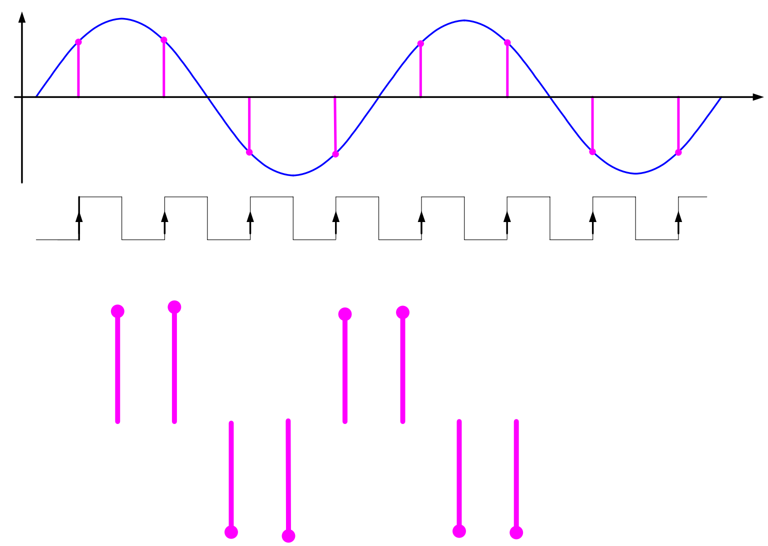
What happens if the requirements for the sampling theorem are not met?

Example:

$f_{SIG}=3/4\ f_{CLK}$



$f_{SIG}=1/4\ f_{CLK}$



Since two different signals have same sampled sequence,
can not uniquely reconstruct the signal from the samples
In this example, the signal at $3/4f_{CLK}$ has been aliased to frequency $1/4f_{CLK}$

Review from Last Time:

Time Quantization

What happens if the requirements for the sampling theorem are not met?

Since two different signals have same sampled sequence, can not uniquely reconstruct the signal from the samples

This makes the samples of a signal that was at a frequency above the Nyquist Rate look like those of a signal that meets the Nyquist Rate requirements

The creation of samples that appear to be of a lower frequency is termed aliasing.

Aliasing will occur if signals are sampled with a clock of frequency less than the Nyquist Rate for the signal.

Time Quantization

What happens if the requirements for the sampling theorem are not met?

Aliasing will occur if signals are sampled with a clock of frequency less than the Nyquist Rate for the signal.

If aliasing occurs, what is the aliasing frequency ?

This calculation is not difficult but a general expression will not be derived at this time. If can be shown that if f is a frequency above the Nyquist rate, then the aliased frequency will be given by the expression

$$f_{ALIASED} = (-1)^{k+1} f + (-1)^{k} \left[\frac{k}{2} + \frac{-1+(-1)^{k}}{4} \right] f_{SAMP} \qquad \text{for} \qquad \frac{k-1}{2} f_{SAMP} < f < \frac{k}{2} f_{SAMP}$$

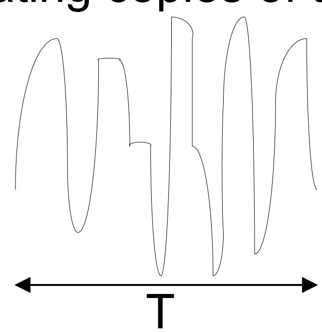
where k is an integer greater than 1 and where f_{SAMP} is the sampling frequency

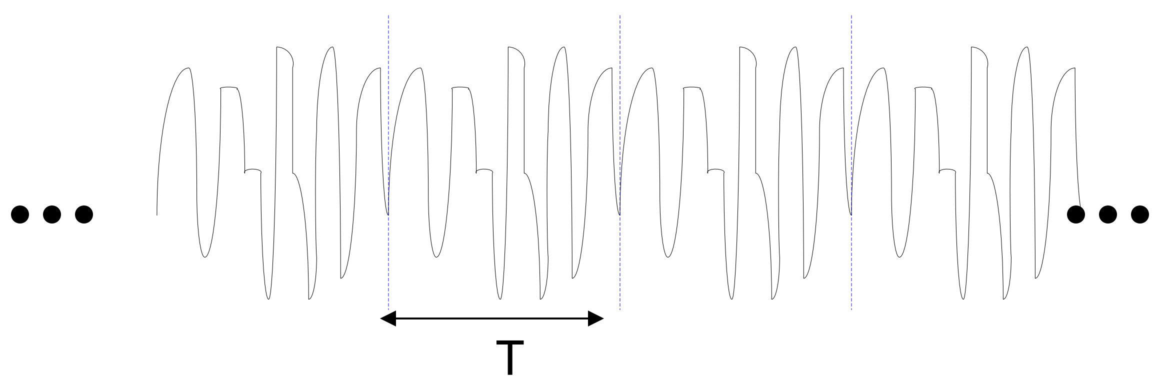
Time Quantization

The sampling theorem and aliasing, another perspective

Previous discussion was based upon a single sinusoid – the implications of sampling apply to much more general waveforms !

Any signal observed for a time interval T can be mapped to a periodic signal of period T by indefinitely repeating copies of the signal observed in the interval T





Time Quantization

The sampling theorem and aliasing, another perspective

Recall if y(t) is periodic with period T, y(t) can be expressed as a Fourier Series

$$y(t)=A_0+\sum_{k=1}^{\infty}A_k\sin(k\omega t+\theta_k)$$

where $\omega=\frac{2\pi}{T}=2\pi f$

A periodic signal y(t) is band-limited to a frequency mf if $A_k=0$ for all k>m

Thus, if y(t) is band-limited to mf, then y(t) can be expressed as

$$y(t)=A_0+\sum_{k=1}^{m}A_k\sin(k\omega t+\theta_k)$$

Time Quantization

The sampling theorem and aliasing, another perspective

If a periodic signal is band-limited to mf, then the Nyquist Rate for the signal is f_{NYQ}=2mf

$$y(t)=A_0+\sum_{k=1}^{m}A_k\sin(k\omega t+\theta_k) \qquad \omega=\frac{2\pi}{T}=2\pi f$$

The Sampling Theorem (for periodic signals)

An exact reconstruction of a continuous-time periodic signal of period T from its samples can be obtained if the signal is the sampled at a frequency that exceeds the Nyquist Rate of the signal.

Furthermore, the signal can be reconstructed by taking 2m+1 consecutive samples and solving the resultant 2m+1 equations for the 2m+1 unknowns <A_0, A_1, …A_m> and <θ_1, θ_2,… θ_m> and then expressing the signal by

$$y(t)=A_0+\sum_{k=1}^{\infty}A_k\sin(k\omega t+\theta_k) \qquad \text{where} \qquad \omega=\frac{2\pi}{T}=2\pi f$$

Time Quantization

How often must a signal be sampled so that enough information about the original signal is available in the samples so that the samples can be used to represent the original signal ?

If the signal is not band-limited, there will be insufficient information gathered in any sampled sequence to completely represent the signal by the sampled sequence

If a signal is band-limited, the signal must be sampled at a rate that exceeds the Nyquist Rate for that signal

The sampling theorem only states that sufficient information is present in the samples if the hypothesis of the theorem is satisfied but does not tell how to reconstruct the signal.

If a signal is not band limited or if it is sampled at a frequency below the Nyquist Rate, higher-frequency components will be aliased into lower frequency regions

If the energy in a signal at frequencies above the effective Nyquist Rate as determined by a sampling clock is small, the aliased high-frequency components will be small as well

Time Quantization

How often must a signal be sampled so that enough information about the original signal is available in the samples so that the samples can be used to represent the original signal ?

Often the information of interest in a signal is band-limited even though the signal is not band limited.

Can this information be extracted by sampling?
(That is, can the signals of interest be reconstructed from an appropriate number of samples?)

Time Quantization

Anti-aliasing Filters

$$y(t) \longrightarrow \boxed{\begin{array}{c}\text{Filter}\\ T(s)\end{array}} \longrightarrow y_F(t)$$

From Laplace Transforms

$$Y_F(s) = Y(s)\,T(s)$$

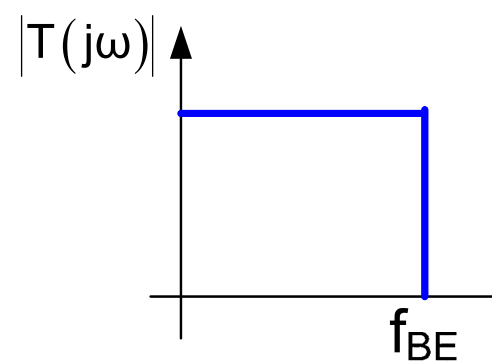
From Fourier Transforms

$$Y_F(\omega) = Y(\omega)\,T(j\omega)$$

From Fourier Series

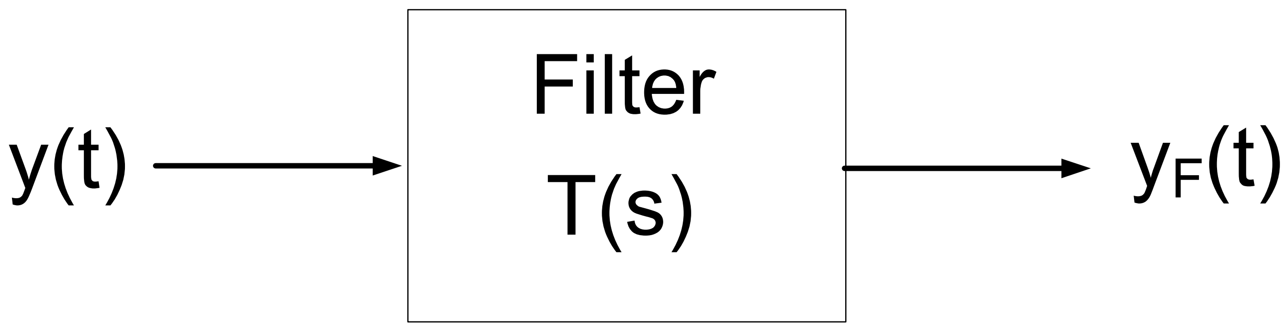
$$y(t) = A_0 + \sum_{k=1}^{\infty} A_k \sin(k\omega t + \theta_k)$$

$$A_{kF} = A_k \left|T(j\omega)\right|$$



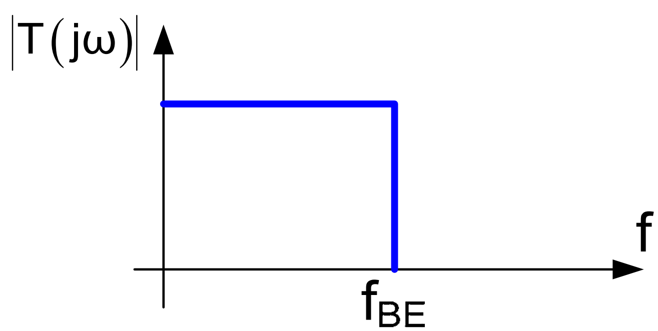
Time Quantization

Anti-aliasing Filters



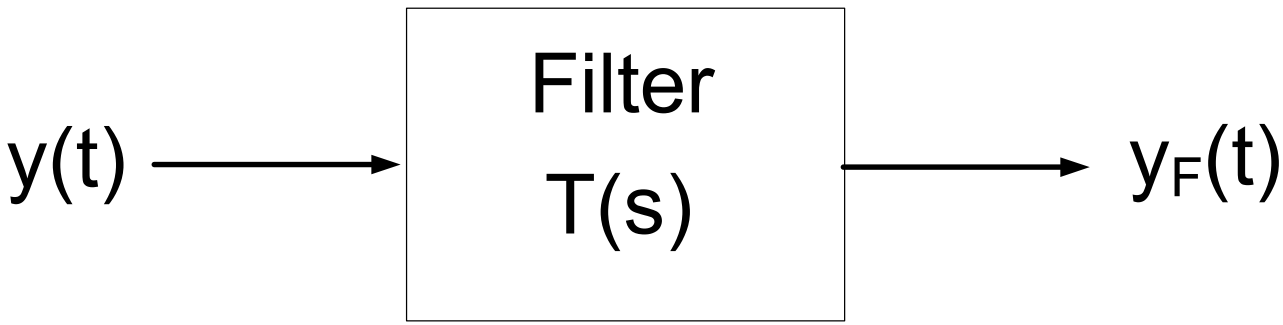
If T(s)=0 for $f > f_{BE}$ then $y_F(t)$ is band-limited

If T(s) is an ideal lowpass function with band edge f_{BE} and y(t) is either not band-limited or band-limited with a signal bandwidth that is larger than f_{BE}, then $y_F(t)$ is band-limited with signal bandwidth f_{BE}.



Time Quantization

Anti-aliasing Filters



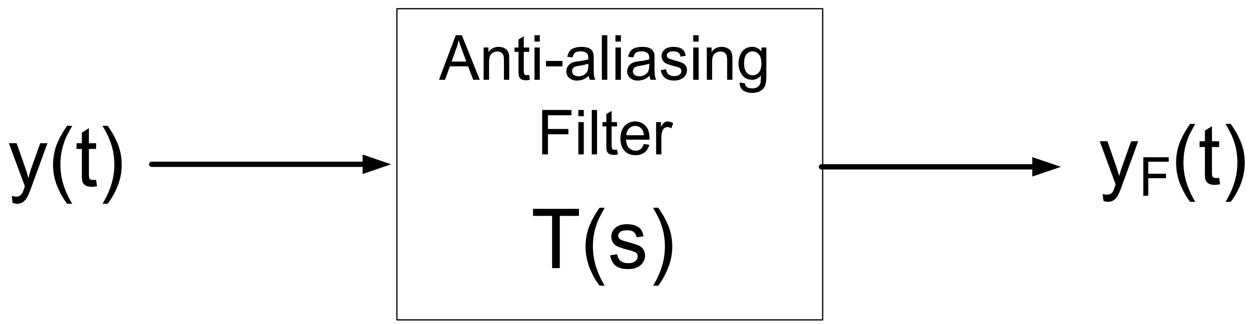
Lowpass filters are widely used to limit the bandwidth of a signal y(t) to the band-edge of the filter before the signal is sampled.

Lowpass filters that are used in this application are termed “Anti-aliasing” filters

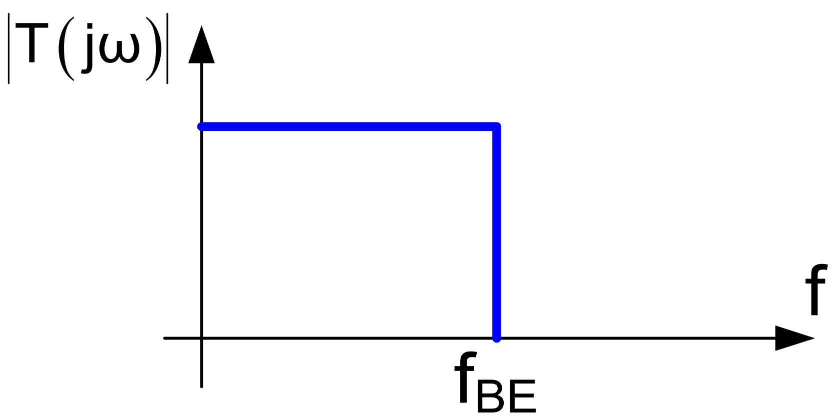
Although the ideal lowpass filter function can not be implemented, lowpass filters with varying degrees of sharpness in the transition are widely available and well-studied. Some filters that are used for anti-aliasing filters include Butterworth, Chebyschev and Elliptic filters of varying order depending upon how Steep of a transition form the passband to the stop band is required..

Time Quantization

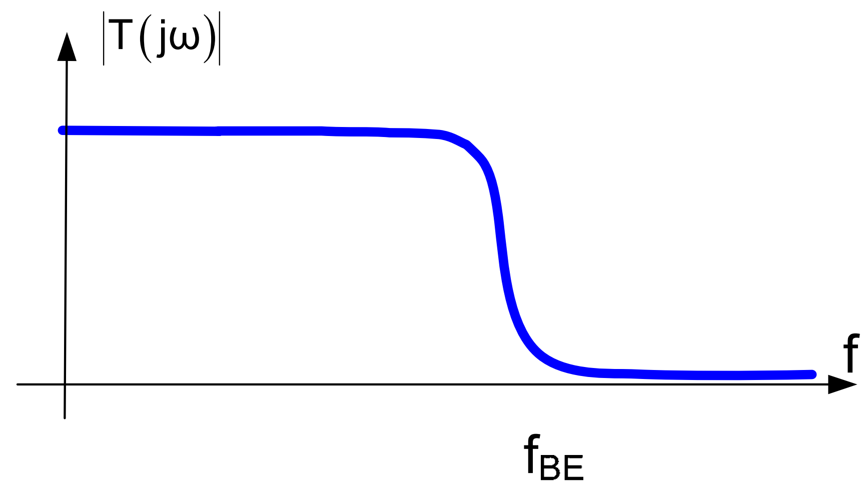
Anti-aliasing Filters



Ideal anti-aliasing filter

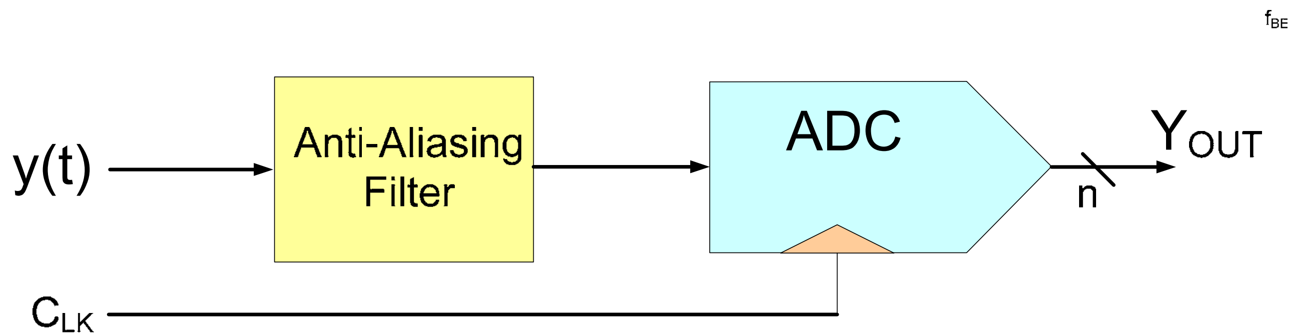


Typical anti-aliasing filter



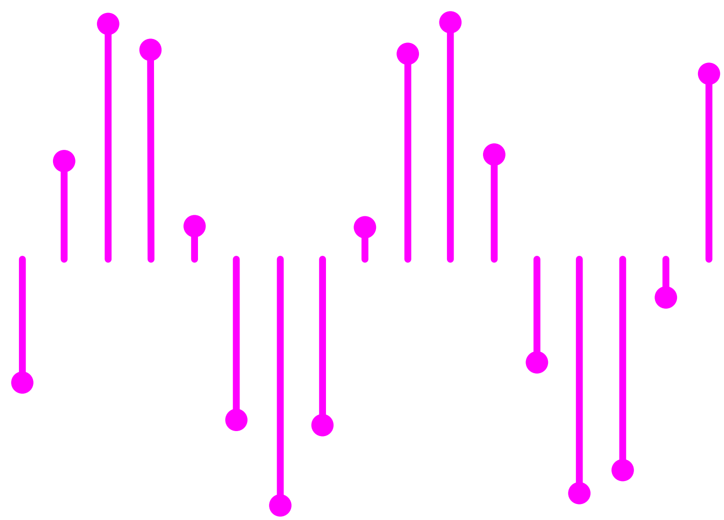
Time Quantization

Typical ADC Environment



Time Quantization

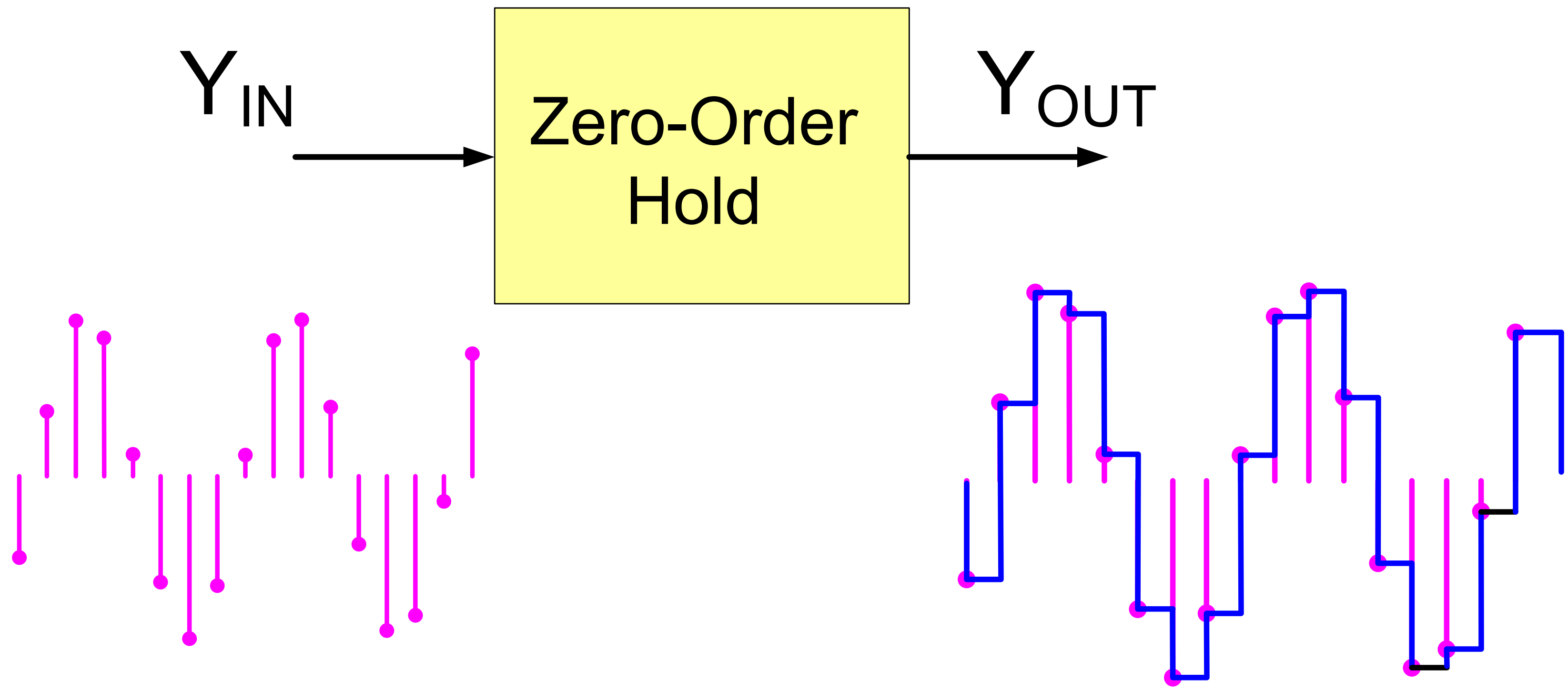
Analog Signal Reconstruction



Boolean sequence represents samples at fixed instances in time

Time Quantization

Analog Signal Reconstruction

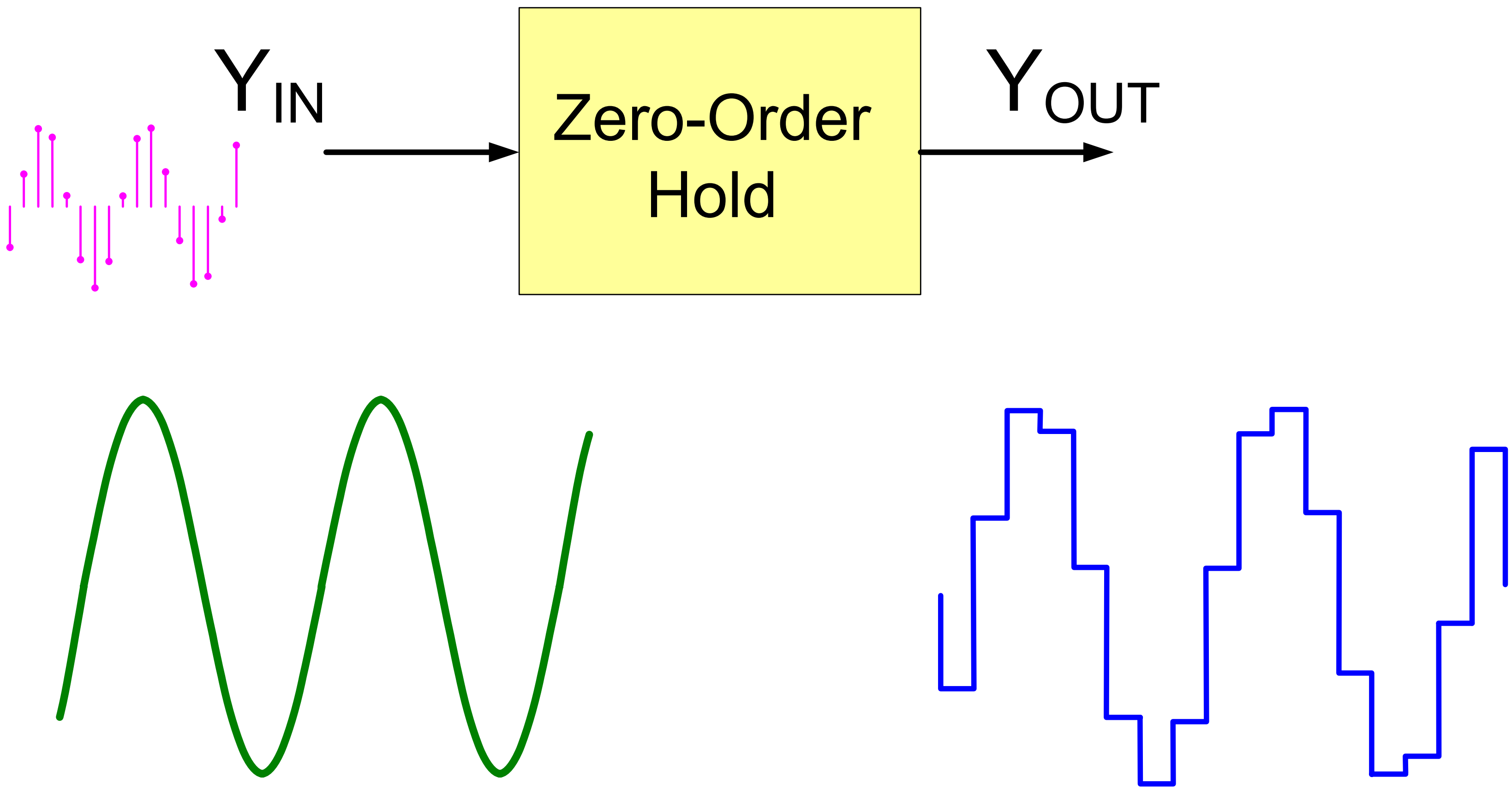


Zero-order Hold can be implemented rather easily with a DAC and other components

Although Sampling Theorem states there is sufficient information in samples to reconstruct input waveform, does not provide simple way to do the reconstruction

Time Quantization

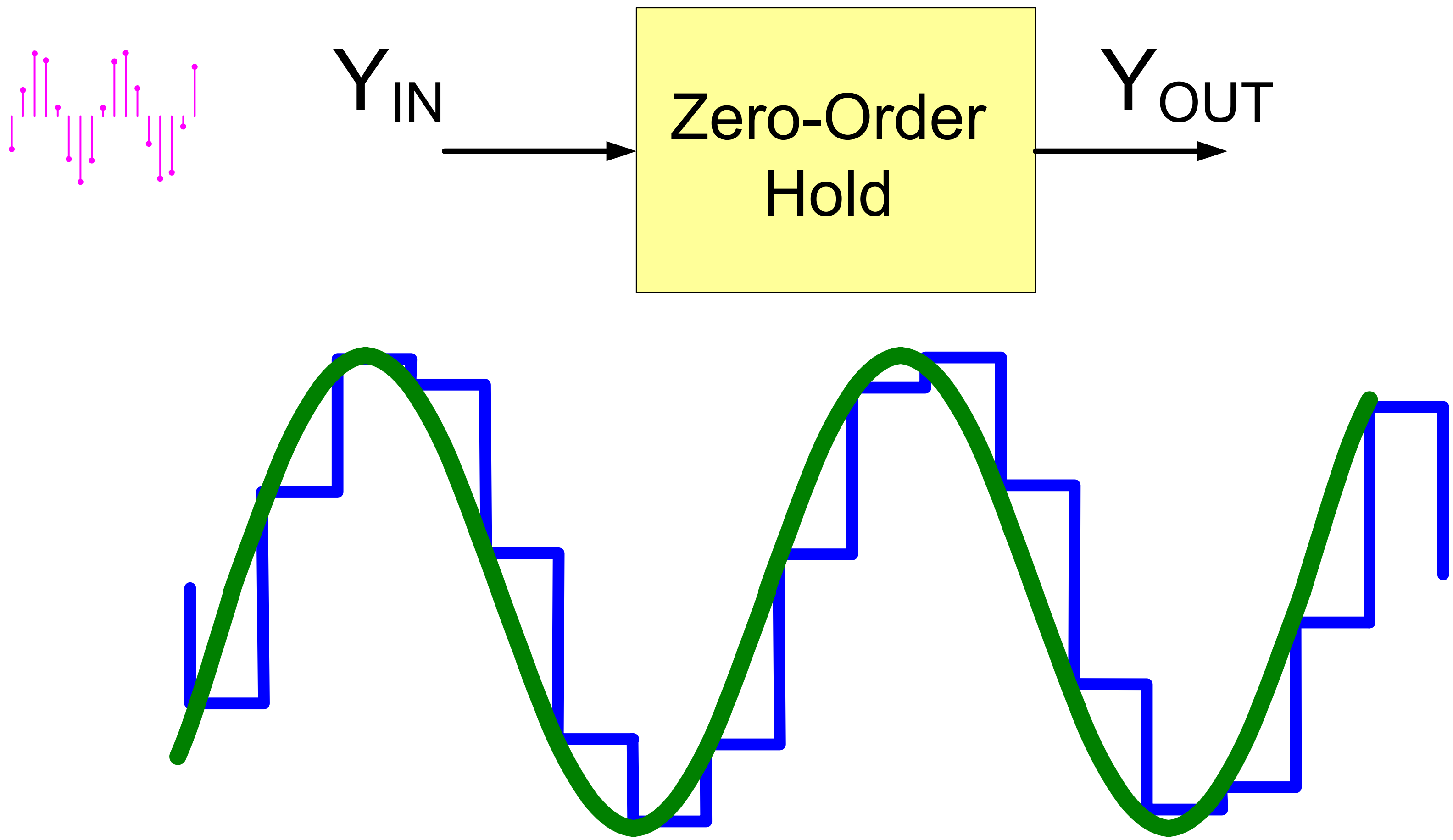
Analog Signal Reconstruction



Although Sampling Theorem states there is sufficient information in samples to reconstruct input waveform, does not provide simple way to do the reconstruction

Time Quantization

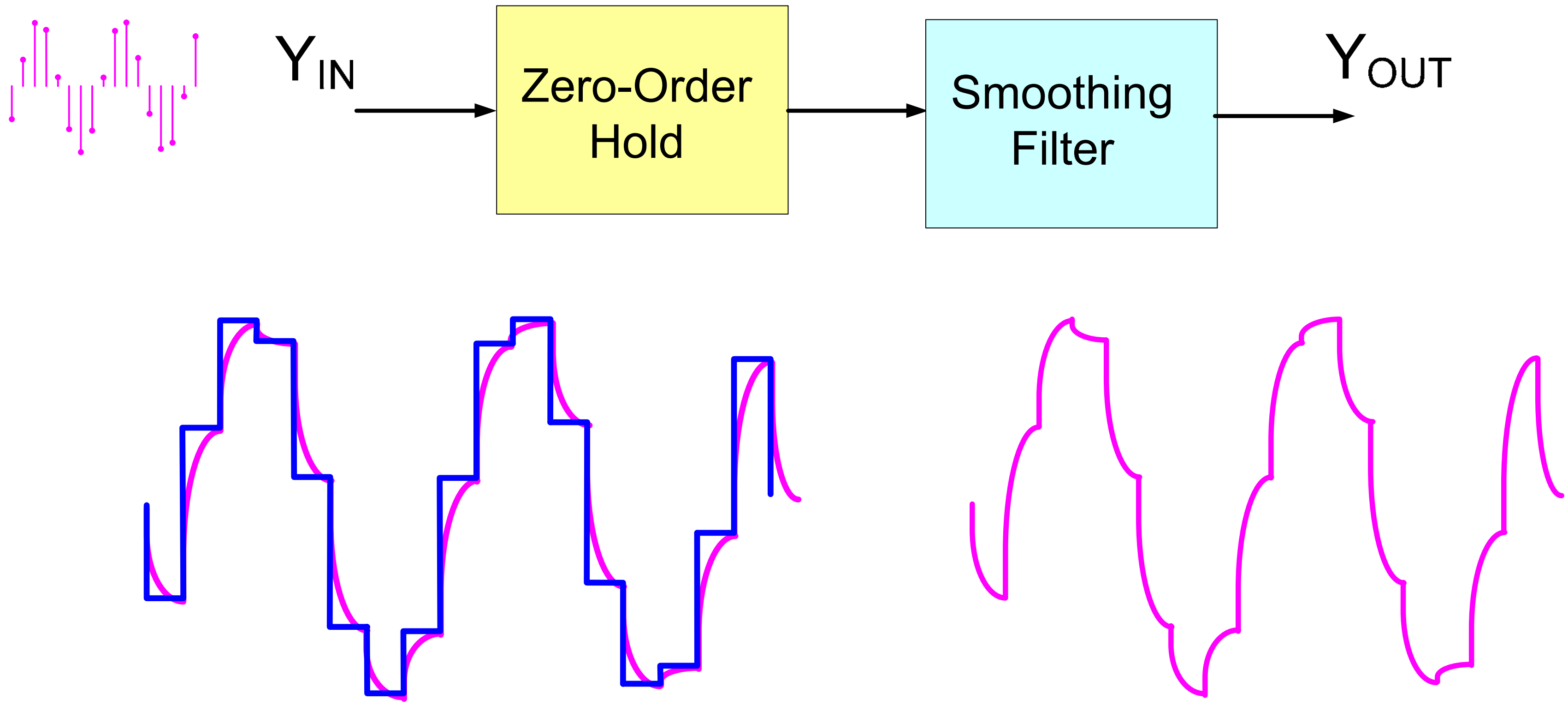
Analog Signal Reconstruction



Zero-order Hold is an approximation to the actual signal that is quite close if highly oversampled but that differs considerably when sampling at near the Nyquist rate

Time Quantization

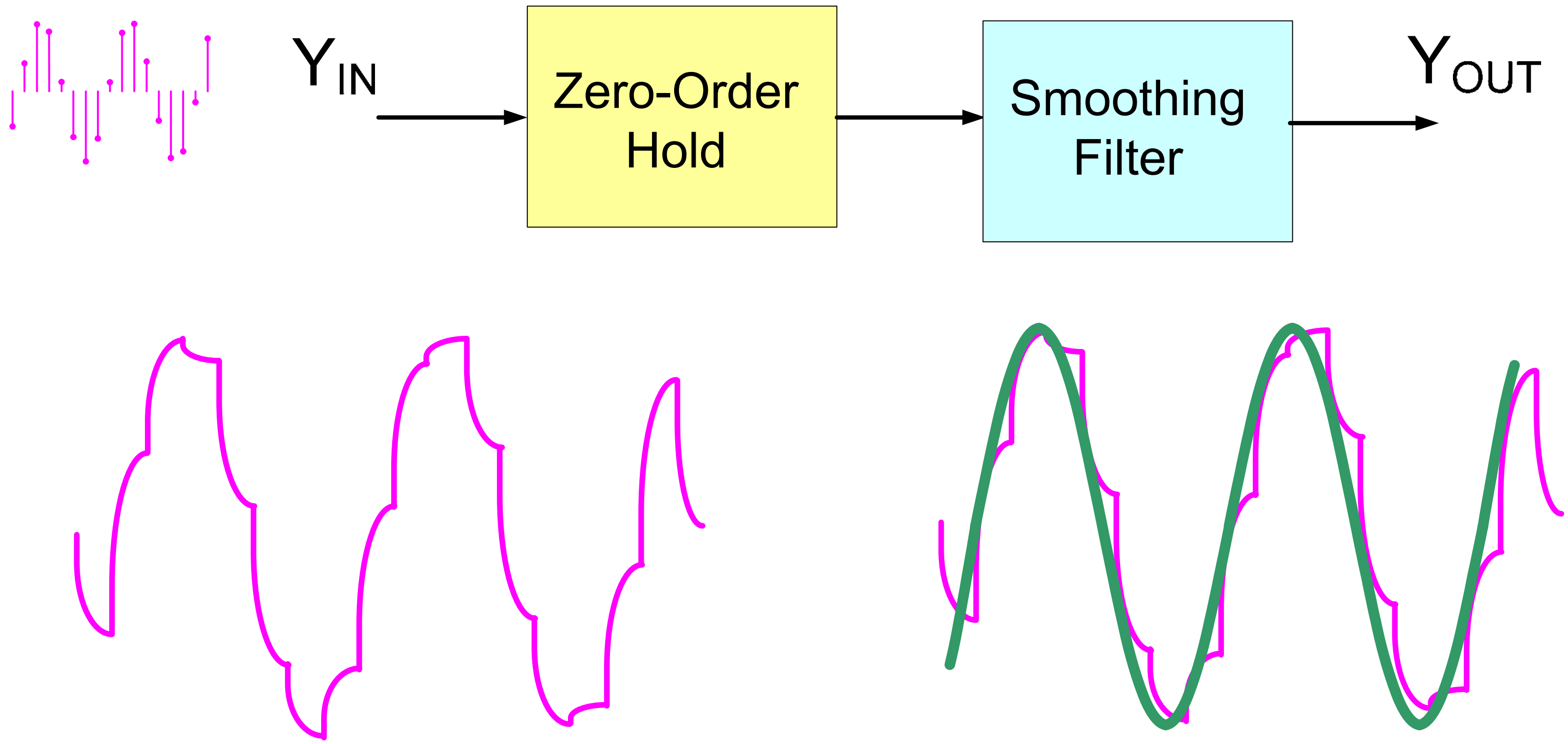
Analog Signal Reconstruction



Smoothing filter removes some of the discontinuities in the output of the zero-order hold

Time Quantization

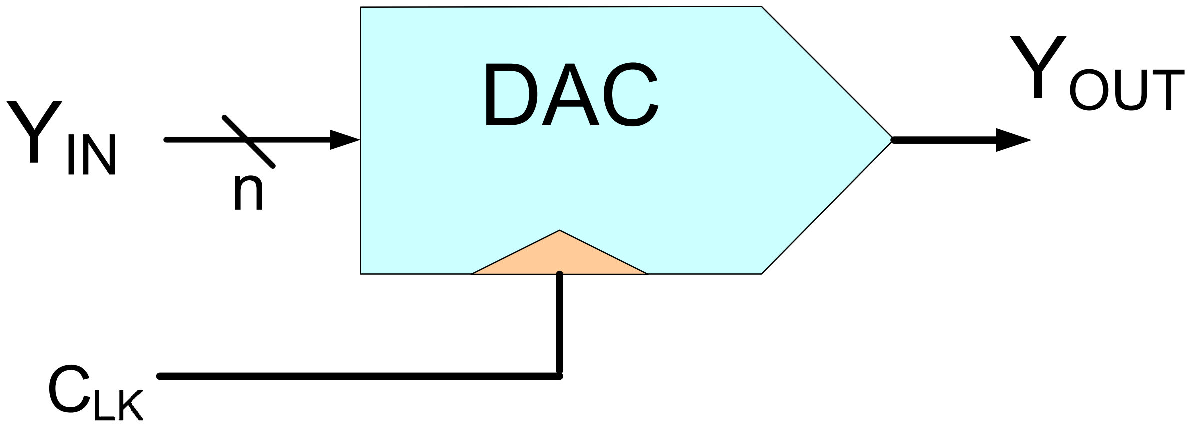
Analog Signal Reconstruction



Smoothing filter removes some of the discontinuities in the output of the zero-order hold

Time Quantization

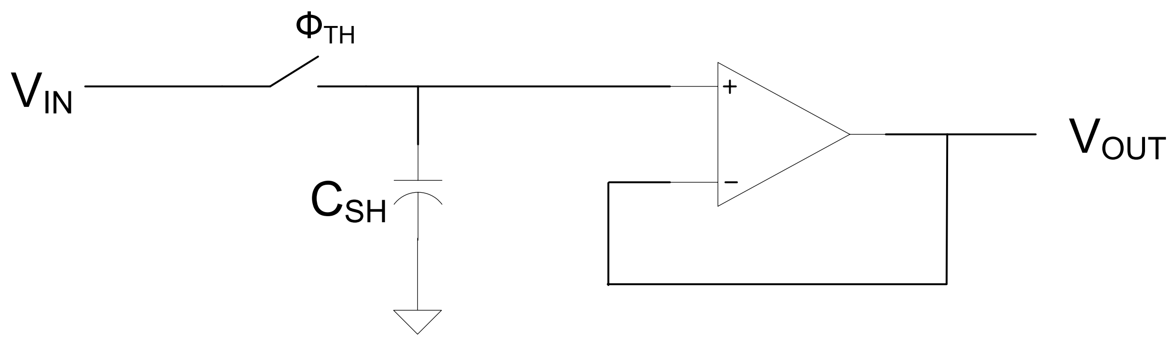
Analog Signal Reconstruction



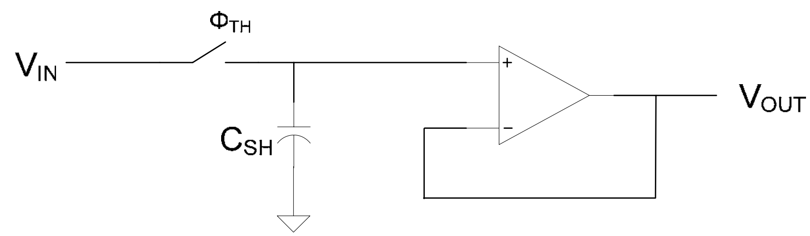
For many DACs, output only valid at some times – e.g. when clock is high

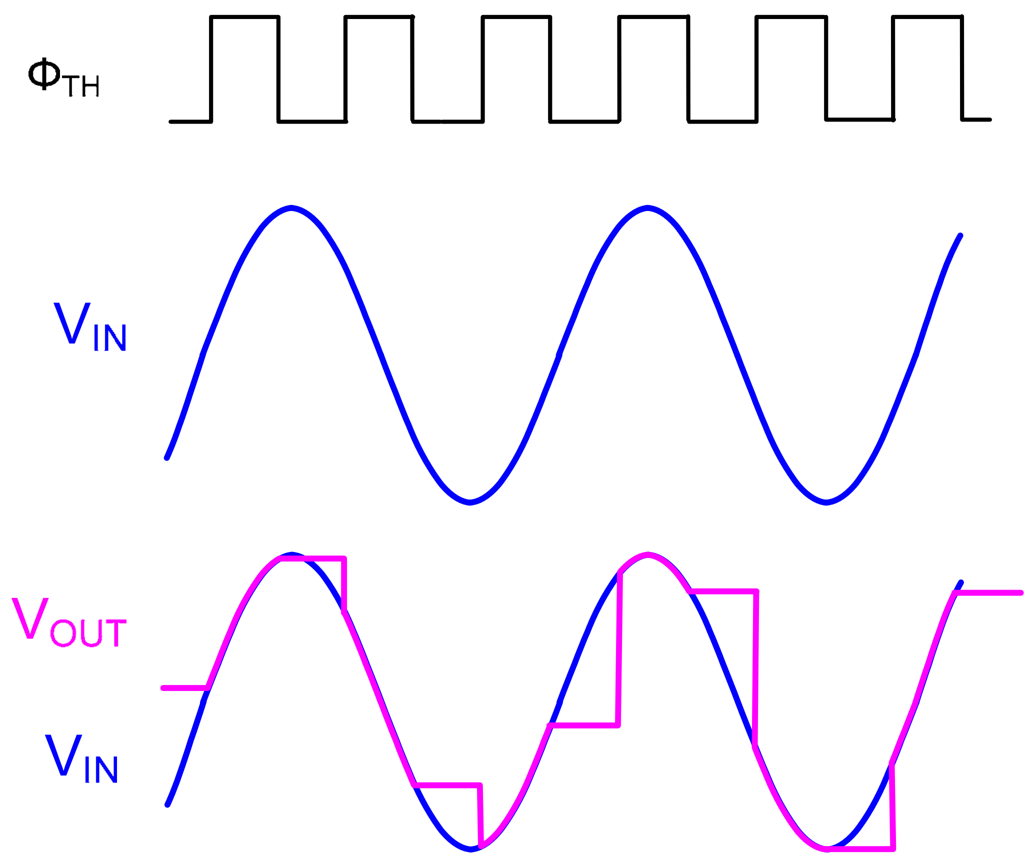
Time Quantization

Track and Hold

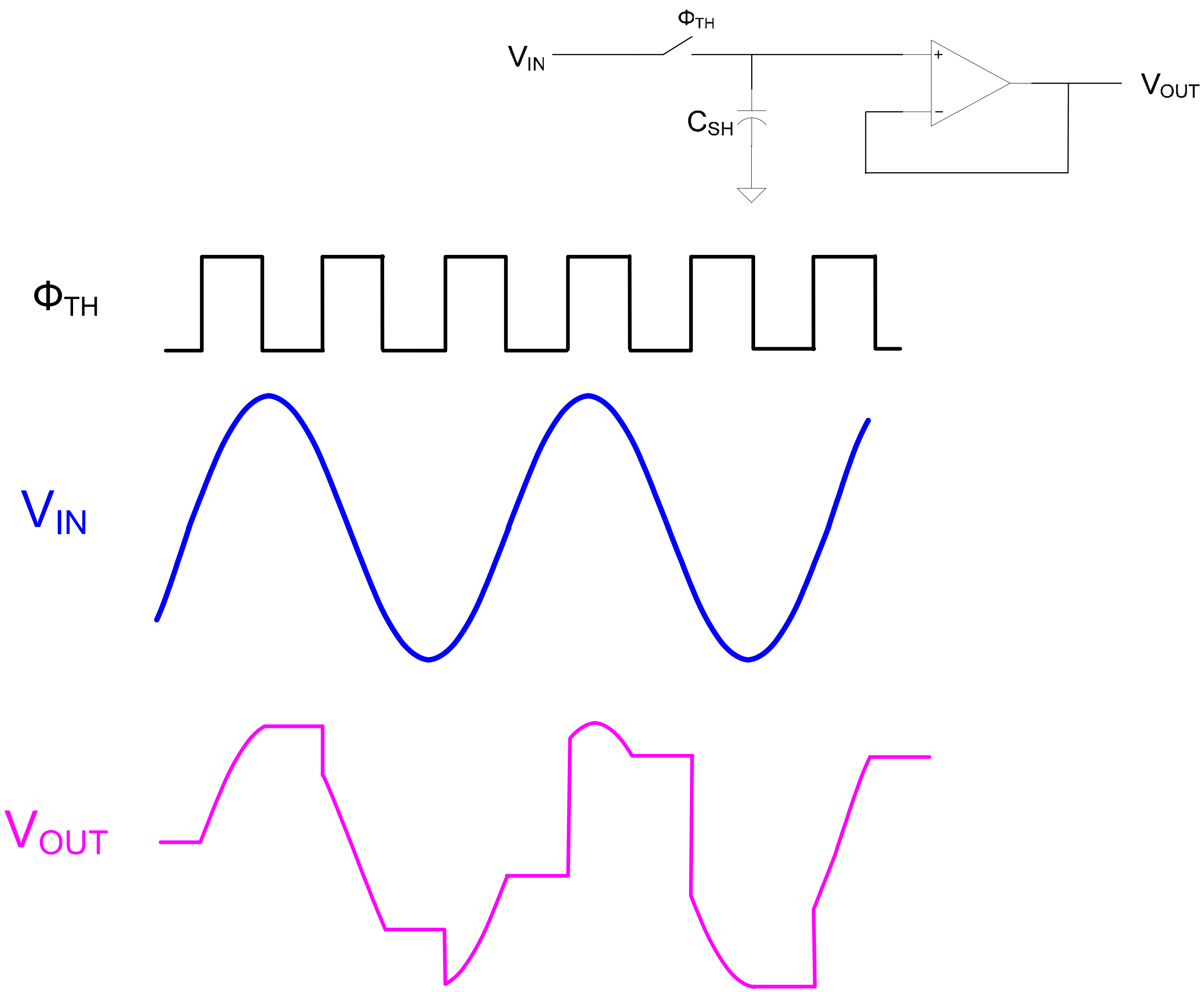


Track and Hold



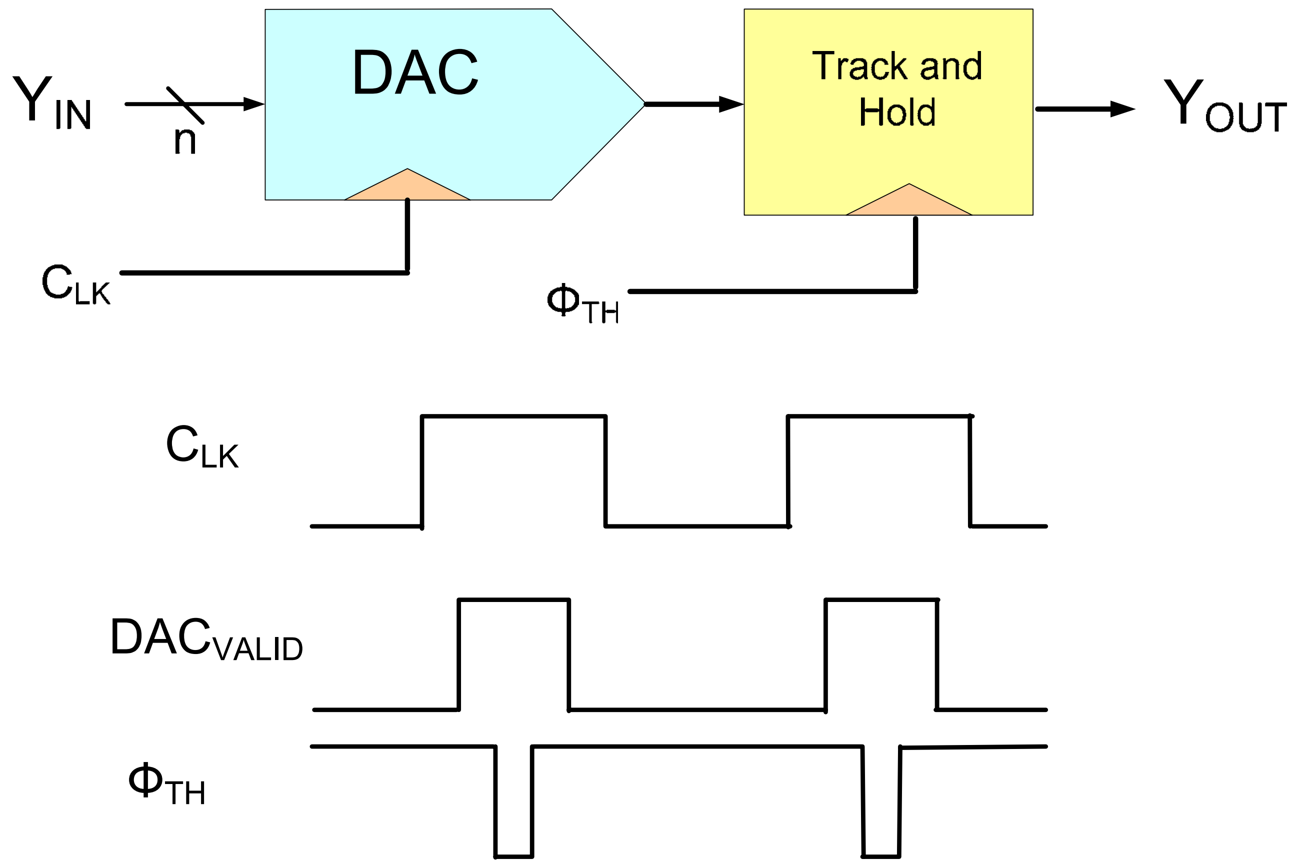


Track and Hold



Time Quantization

Analog Signal Reconstruction



- Also useful for more general DAC applications
- T/H may be integrated into the DAC

Time Quantization

Sampling Theorem

- Aliasing
- Anti-aliasing Filters
- Analog Signal Reconstruction

Engineering Issues for Using Data Converters

Inherent with Data Conversion Process

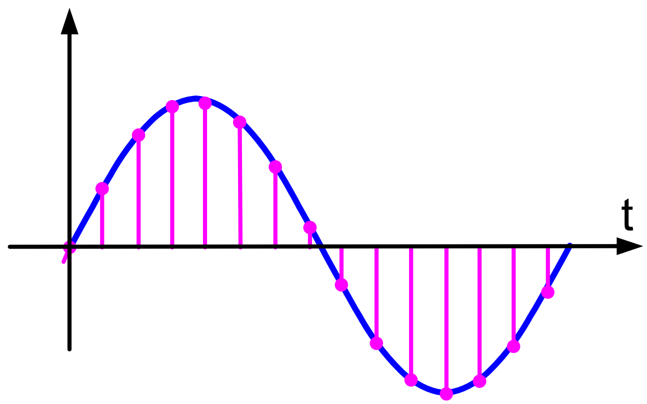
- Time Quantization

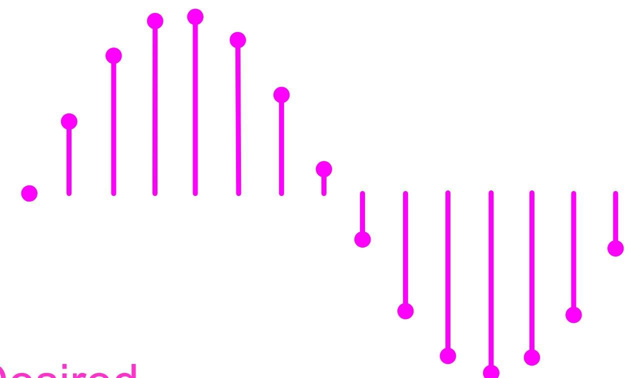
→ Amplitude Quantization

How do these issues ultimately impact performance ?

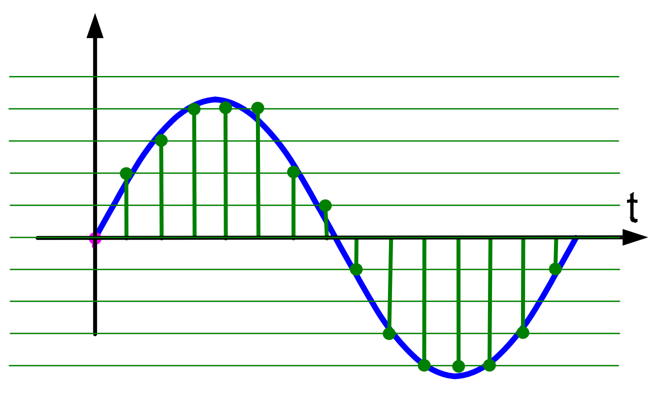
Amplitude Quantization

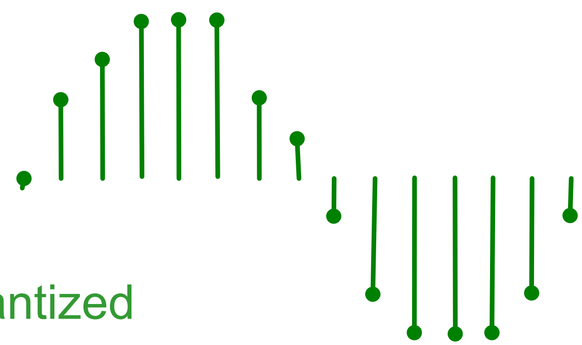
Analog Signals at output of DAC are quantized
Digital Signals at output of ADC are quantized





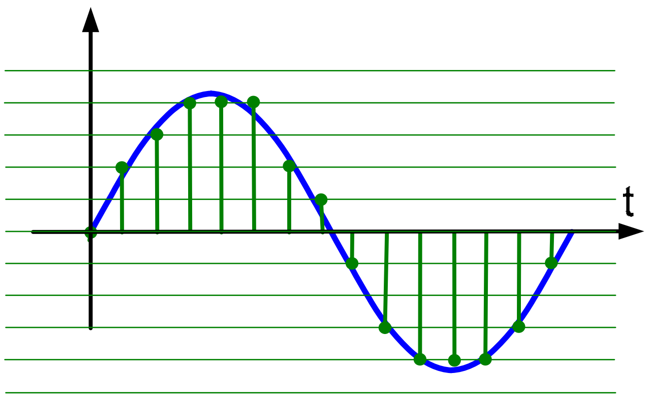
Desired

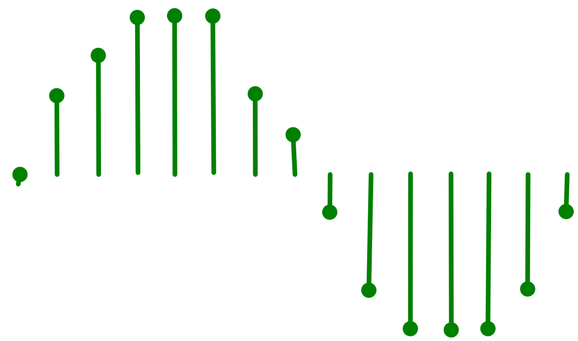




Quantized

Amplitude Quantization





Amplitude quantization introduces errors in the output

About all that can be done about quantization errors is to increase the resolution and this is the dominant factor that determines the required resolution in most applications

Quantization errors are present even in ideal data converters !

Noise and Distortion

Unwanted signals in the output of a system are called noise.

There are generally two types of unwanted signals in any output

- Distortion
- Signals coming from some other sources

Amplitude Quantization

Unwanted signals in the output of a system are called noise.

Distortion

- Smooth nonlinearities
- Frequency attenuation
- Large Abrupt Nonlinearities

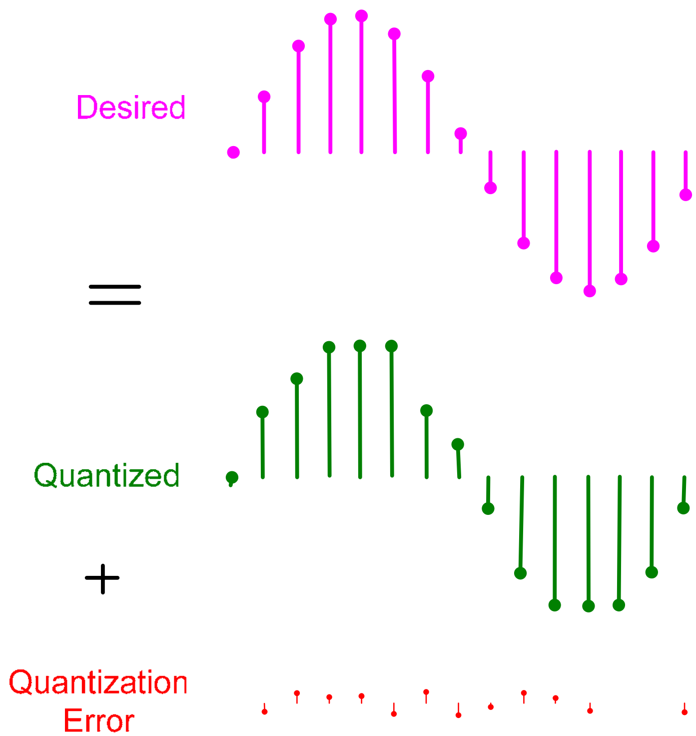
Signals coming from other sources

- Movement of carriers in devices
- Interference from radiating sources
- Interference from electrical coupling

Amplitude Quantization

Any unwanted signal in the output of a system is called <u>noise</u>

- Amplitude quantization introduces errors in the output
  - – quantization error called noise



Amplitude Quantization

How big is the quantization “noise” characterized?